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MODIFICATION OF THE FRACTURE CRITERION FOR V-SHAPED NOTCHES (PLANE PROBLEM). RELATIONSHIP BETWEEN TOUGHNESS AND STRENGTH AND STRUCTURAL PARAMETERS

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A fracture criterion of the type of the Neuber–Novozhilov criterion is proposed to describe the fracture in the vicinity of the tip of a V-shaped notch under tensile and shear loading. In the proposed criterion, the limits of averaging of the stresses along the notch axis depend on the presence, location, and size of the initial defects in the material. The crystal lattice parameter of the initial material is chosen for the characteristic linear size. For a V-shaped notch subjected to tension and shear, simple equations are obtained that relate the stress intensity factors for the modified singularity coefficients, the singularity coefficients themselves, and the theoretical tensile and shear strengths of a single crystal of the material taking into account the damage to the material in the vicinity of the notch tip. The equations obtained allow a passage to the limit from a notch to a crack. It is shown that the classical critical stress intensity factor used in the strength analysis of cracked solids is not a material constant.

Key words: *fracture, brittle fracture criteria, crack, V-shaped notch, stress intensity factor.*

Introduction. In classical fracture mechanics, the force and deformation fracture criteria are intended for application to domains with cracks. Attempts to use these criteria directly in the fracture analysis of bodies with V-shaped stress concentrators fail as a rule. The Neuber–Novozhilov approach [1, 2] allows one to describe the fracture of cracked bodies with a structural hierarchy [3–5] under loads corresponding to three classical modes of cracks. The discrete integral criteria are based on the concepts of classical fracture mechanics (solid mechanics) and solid state physics [6, 7] related to the crystal structure of single crystals. If the real spatial arrangement of atoms in a single crystal is taken into account, cracks in it cannot be modeled by bilateral notches. Even in the two-dimensional case, it makes sense to consider V-shaped notches with the tip angle dependent on the characteristics of the crystal lattice. The peculiarity of fracture problems for bodies with sharp notches is discussed in [8, 9]. In the vicinity of a V-shaped notch, the stress fields consist of regular and singular components and the singularity coefficient depends on the tip angle [10] and coincides with the singularity coefficient of the stress field near a crack only in the limiting case where a V-shaped notch becomes a bilateral notch (crack).

The drawbacks of classical crack theory, in which material structure is ignored, have stimulated the construction of multiscale strength criteria taking into account the structural hierarchy: macrolevel (standard reinforced specimen), mesolevel (regular granularity of materials), and microlevel (particular structure of the atomic lattice in the vicinity of the crack tip). Panasyuk et al. [11] studied the dependence of the critical stress intensity factor (SIF) on the standard mechanical characteristics of materials with allowance for material structure. Kornev [4] proposed consistent discrete integral strength criteria for mode I cracks in solids containing a hierarchy of embed-

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ded structures whose linear sizes differ by two orders of magnitude and can vary from 10^{-7} to 10^2 cm. Multiscale shear strength criteria for rock blocks with block-hierarchical structure are considered in [5]. In [12], a generalized sufficient discrete integral strength criterion describing the prefracture zone for mode I cracks in structured media was constructed for elastoplastic materials. In addition, exact and approximate equations were obtained that relate critical parameters, the theoretical strength of granular materials, grain size, and the parameters characterizing the averaging interval and the damage to the initial and plastically deformed material.

In the last decade, the shape and size of notches or inner angles in structural members have been studied more extensively than the crack shape and size (see, e.g., papers by Seweryn [13–15], Carpinteri [16], and Dunn [17, 18]). The effect of the stress–strain state near V-shaped notches on the strength of welded joints is considered in [19]. The stress field near a V-shaped notch for antiplane deformation is described in [20]. In the present paper, the applicability of the Novozhilov criterion to determining the fracture strength of solids with V-shaped notches is analyzed using as an example an elastic half-plane with an edge V-notch subjected to combined tensile and shear loading at infinity.

Stresses in the Vicinity of the Tip of a V-Shaped Notch. Let a plate under plane-strain or plane-stress conditions be bounded by two intersecting planes so that the region studied is an infinite dihedral angle 2α (Fig. 1). We study the stress field in the vicinity of the notch tip caused by tensile and shear stresses applied at infinity. We introduce Cartesian coordinates Oxy and polar coordinates $Or\theta$. It is assumed that the body and load are symmetric about the notch axis. Then, because of the symmetry of the problem, the maximum stresses occur on the notch axis. In the vicinity of the notch tip, the linear solution for the tensile stress $\sigma_\theta(r,\theta)$ and the shear stress $\tau_{r\theta}(r,\theta)$ on the notch axis $\theta = 0$ can be written with accuracy up to terms of higher order of smallness as follows:

$$\sigma_\theta(r,0) \simeq \sigma_\infty + K_{\rm I} r^{\lambda_1 - 1}/\sqrt{2\pi} \qquad (1)$$

for tension and

$$\tau_{r\theta}(r,0) \simeq \tau_\infty + K_{\rm II} r^{\lambda_2 - 1}/\sqrt{2\pi} \qquad (2)$$

for pure shear. Here $\sigma_\infty = \text{const}$ and $\tau_\infty = \text{const}$ are the characteristic stresses, $K_{\rm I}$ is the generalized tensile SIF for the singular component r^{λ_1-1}, $K_{\rm II}$ is the generalized pure-shear SIF for the singular component r^{λ_2-1}, and $\lambda_1 = \lambda_1(\alpha)$ and $\lambda_2 = \lambda_2(\alpha)$ are roots of the characteristic equations [10]

$$\sin 2\lambda_1\alpha + \lambda_1 \sin 2\alpha = 0 \qquad (3)$$

and

$$\sin 2\lambda_2\alpha - \lambda_2 \sin 2\alpha = 0 \qquad (4)$$

for tension and shear, respectively. The characteristic stresses σ_∞ and τ_∞ are determined from the constructed stress fields $\sigma_\theta(r,\theta)$ and $\tau_{r\theta}(r,\theta)$.

Figure 2 shows the solutions of Eqs. (3) and (4) versus the angle α (the dashed curve refers to the solutions of these equations that do not contain a singular component). For $\alpha = \pi$ (crack) and $\alpha = \pi/2$ (half-plane), the roots λ_1 and λ_2 coincide. For any root $\lambda < 0$, the displacements at the point $r = 0$ tend to infinity and, hence, this case is not considered. If $\lambda = 0$, the total strain energy in any circle $r < R$ enclosing the notch tip is unlimited, which is physically impossible. In view of these considerations, all roots $\lambda \le 0$ should be eliminated from the consideration. We note that for $\alpha < 2.252$, the stresses $\tau_{r\theta}$ are limited in the vicinity of the notch tip since $\lambda_2 > 1$. Moreover, $\lambda_1 < \lambda_2$ in the entire range of the angle α. It follows that for $\pi/2 < \alpha < \pi$, the degree of singularity of the stresses produced near the notch tip by tensile loading is higher than that of the stresses due to shear at infinity. We consider two cases: $\lambda_1 > 1/2$ and $\lambda_2 > 1/2$ for $\alpha < \pi$ and $\lambda_1 = \lambda_2 = 1/2$ for $\alpha = \pi$ (bilateral cut). For the crack, the generalized SIFs $K_{\rm I}$ and $K_{\rm II}$ become the classical SIFs $K_{\rm I}^0$ and $K_{\rm II}^0$ for a sharp crack. Only in the last case can methods of classical fracture mechanics for cracked solids be applied. In the first case ($\alpha < \pi$), the singularity of the stress field is smaller than that of the stress field at the crack tip; therefore, the classical approach is not applicable to the strength analysis of bodies with V-shaped notches. Thus, we have $1/2 < \lambda_1 < 1$ and $1/2 < \lambda_2 < 1$ for $\pi/2 < \alpha < \pi$. For $\alpha = \pi/2$, we obtain $\lambda_1 = \lambda_2 = 1$; in this case, a singular component is absent.

Brittle Fracture Criterion for Bodies with V-Shaped Notches. We consider a single crystal with a V-shaped notch whose tip angle depends on the crystal-lattice characteristics. We confine ourselves to a two-dimensional case. It is assumed that there are vacancies ahead of the notch tip. Figure 3 gives a schematic diagram

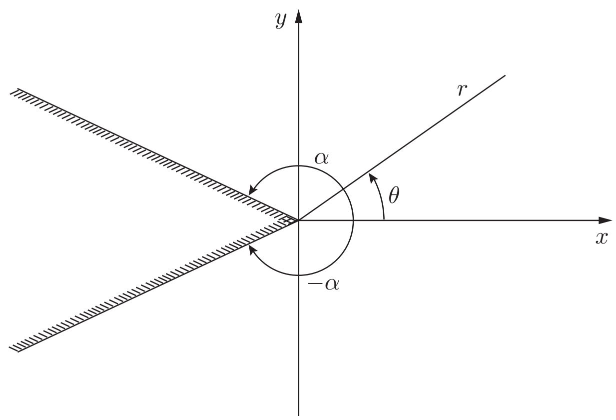

Fig. 1

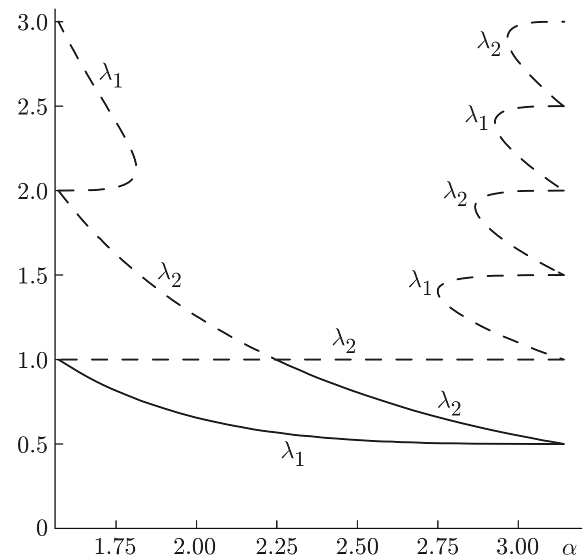

Fig. 2

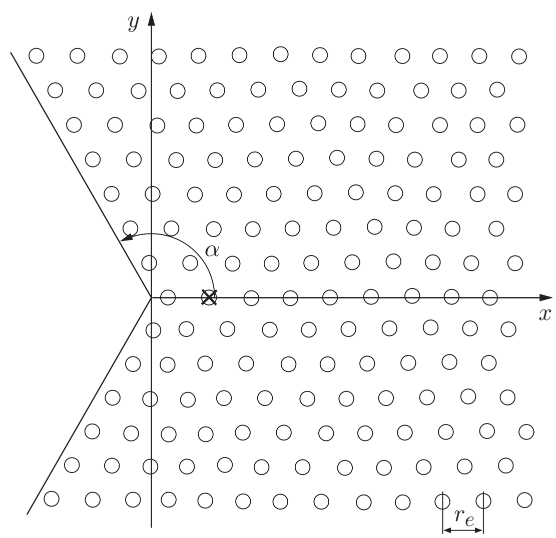

Fig. 3

of such a close-packed layer of atoms with a vacancy (the atoms are shown by circles and the vacancy is denoted by a cross; r_e is the interatomic distance and $\alpha = 2\pi/3$).

For the weakest monatomic layer, the following discrete integral criterion of brittle strength (for the two-dimensional case) is proposed for V-shaped notches under plane-strain or plane-stress conditions:

$$\frac{1}{kr_e}\int\limits_0^{nr_e} \sigma_\theta(r,0)\,dr \le \sigma_m \tag{5}$$

for tension on the notch axis and

$$\frac{1}{kr_e}\int\limits_0^{nr_e} \tau_{r\theta}(r,0)\,dr \le \tau_m \tag{6}$$

for pure shear on the notch axis. In formulas (5) and (6), $\sigma_\theta(r,0)$ are the tensile stresses on the notch axis, $\tau_{r\theta}(r,0)$ are the shear stresses on the notch axis (these stresses act within the thickness of the single crystal), n and

k are integers ($n \geq k$, where k is the number of interatomic bonds), nr_e is the averaging interval ($n = 2$ and $k = 1$ for the case shown in Fig. 3), σ_m and τ_m are the theoretical (ideal) tensile and shear strengths of the single crystal for the plane $\theta = 0$, respectively.

Substituting relations (1) and (2) into (5) and (6), respectively, after some transformations we obtain the following estimates of the generalized SIFs $K_{\rm I}$ and $K_{\rm II}$ for a V-shaped notch in the presence of vacancies on its axis in the plane problem:

$$\frac{K_{\rm I}}{\lambda_1\sqrt{2\pi}(nr_e)^{1-\lambda_1}}\,\frac{1}{\sigma_\infty} \leq \frac{\sigma_m}{\sigma_\infty}\,\frac{k}{n} - 1 \tag{7}$$

for tension and

$$\frac{K_{\rm II}}{\lambda_2\sqrt{2\pi}(nr_e)^{1-\lambda_2}}\,\frac{1}{\tau_\infty} \leq \frac{\tau_m}{\tau_\infty}\,\frac{k}{n} - 1 \tag{8}$$

for pure shear. As $\alpha \to \pi$, we obtain $\lambda_1 = \lambda_2 = 1/2$, and estimates (7) and (8) become estimates for the classical SIFs $K_{\rm I}^0$ and $K_{\rm II}^0$ for a sharp crack:

$$2K_{\rm I}^0/(\sigma_\infty\sqrt{2\pi nr_e}\,) \leq (\sigma_m/\sigma_\infty)(k/n) - 1 \tag{9}$$

for tension and

$$2K_{\rm II}^0/(\tau_\infty\sqrt{2\pi nr_e}\,) \leq (\tau_m/\tau_\infty)(k/n) - 1 \tag{10}$$

for pure shear. For $K_{\rm I}^0 = K_{{\rm I}c}^0$ and $K_{\rm II}^0 = K_{{\rm II}c}^0$, inequalities (9) and (10) become equalities ($K_{{\rm I}c}^0$ and $K_{{\rm II}c}^0$ are the critical mode I and II SIFs in classical fracture mechanics, respectively). According to the data of [21], we obtain $K_{\rm I}^0 = \sigma_\infty\sqrt{\pi l_{nk}}$ and $K_{\rm II}^0 = \tau_\infty\sqrt{\pi l_{nk}}$ for an internal crack and $K_{\rm I}^0 = 1.1215\sigma_\infty\sqrt{\pi l_{nk}}$ and $K_{\rm II}^0 = 1.1215\tau_\infty\sqrt{\pi l_{nk}}$ for an edge crack ($2l_{nk}$ and l_{nk} are the lengths of the internal and edge cracks, respectively); therefore, the critical lengths of these cracks $2l_{nk}^c$ and l_{nk}^c satisfy the equalities

$$\frac{2l_{nk}^c}{r_e} = \Big(\frac{\sigma_m}{\sigma_\infty} - \frac{n}{k}\Big)^2\frac{k^2}{n}, \qquad 2.52\,\frac{l_{nk}^c}{r_e} = \Big(\frac{\sigma_m}{\sigma_\infty} - \frac{n}{k}\Big)^2\frac{k^2}{n}; \tag{11}$$

$$\frac{2l_{nk}^c}{r_e} = \Big(\frac{\tau_m}{\tau_\infty} - \frac{n}{k}\Big)^2\frac{k^2}{n}, \quad 2.52\,\frac{l_{nk}^c}{r_e} = \Big(\frac{\tau_m}{\tau_\infty} - \frac{n}{k}\Big)^2\frac{k^2}{n}. \tag{12}$$

Obviously, relations (11) and (12) allow one to pass to the limit as $K_{\rm I}^0 \to 0$, $K_{\rm II}^0 \to 0$, and $l_{nk} \to 0$. In the absence of microdefects (vacancies) and macrodefects (cracks) in a specimen, we obtain $n = k = 1$ and $l_{nk} = 0$, respectively. In this case, the theoretical strength σ_m or τ_m of an ideal crystalline material is reached.

It should be noted that there exist exact limiting relations [21] that link the SIFs of sharp cracks and the stress-concentration coefficients at the tip of a narrow notch. The stress-concentration coefficients have always been related to the geometry of the problem studied [1], and the critical SIFs $K_{{\rm I}c}^0$ and $K_{{\rm II}c}^0$ of cracks are regarded in classical fracture mechanics as material constants.

Generalization for Structured Media. The brittle-fracture criterion proposed above for a single crystal with a V-shaped notch can be extended to bodies with an hierarchy of regular structures.

Following [4], we consider a solid body which contains p embedded regular structures such that their linear sizes r_i ($i = 1, 2, \ldots, p$) are ordered as follows: $r_i \gg r_{i+1}$ and each linear size differs from the next one by at least two orders of magnitude. We introduce a family of brittle-strength discrete integral criteria consistent for each structure:

$$\frac{1}{k_i r_i}\int\limits_0^{n_i r_i} \sigma_y^{(i)}(x_i, 0)\,dx_i \leq \sigma_m^{(i)}$$

for tension and

$$\frac{1}{k_i r_i}\int\limits_0^{n_i r_i} \tau_{xy}^{(i)}(x_i, 0)\,dx_i \leq \tau_m^{(i)}$$

for pure shear. Here $\sigma_y^{(i)}$ and $\tau_{xy}^{(i)}$ are the normal and shear stresses on the notch axis, respectively, $O_i x_i y_i$ are Cartesian coordinate systems with origins at the apices of notches of various scales, n_i and k_i are integers ($n_i \geq k_i$, where k_i is the number of active bonds acting at the notch tip of the ith structure), $n_i r_i$ is the averaging interval, and $\sigma_m^{(i)}$ and $\tau_m^{(i)}$ are the theoretical tensile and shear strengths of the material of the ith structure, respectively. In each structure $i = 1, 2, \ldots, p$, the number of structural units is constant and equal to one force parameter ($\sigma_m^{(i)}$ or $\tau_m^{(i)}$), one structural parameter r_i, where r_i is the diameter of the "grain" of the structure considered, and the averaging and defect parameters n_i and k_i, respectively, for $k_i < n_i$. Since we consider regular structures, the parameters indicated above are constant for each structure $i = 1, 2, \ldots, p$, but for different structures, they may differ substantially not only in the linear size $r_i \gg r_{i+1}$.

The stresses $\sigma_y^{(i)}$ and $\tau_{xy}^{(i)}$ on the notch axis can be calculated after the solution of the corresponding linear elastic problems for specified loads (p is the number of these problems). Expressing solutions (1) and (2) for the stresses on the notch axis in terms of $K_{\rm I}^{(i)}$ and $K_{\rm II}^{(i)}$, we can write

$$\sigma_y^{(i)}(x_i, 0) \simeq \sigma_\infty^{(i)} + K_{\rm I}^{(i)} x_i^{\lambda_1 - 1}/\sqrt{2\pi},$$

$$\tau_{xy}^{(i)}(x_i, 0) \simeq \tau_\infty^{(i)} + K_{\rm II}^{(i)} x_i^{\lambda_2 - 1}/\sqrt{2\pi}$$

and obtain estimates for the generalized SIFs $K_{\rm I}^{(i)}$ and $K_{\rm II}^{(i)}$ similar to (7) and (8).

Various approaches to determining structural levels and structural parameters are available. McClintock and Irwin [22] suggested five basic structural levels — from macro-objects to microstructures for metals: 10^2, 1, 10^{-2}, 10^{-4}, and 10^{-6} mm and four additional levels; for a specimen 1 mm thick, the first macrostructure corresponds to plane stresses, the last three microstructures correspond to plane strains, and the second microstructure occupies an intermediate position. Neuber [1] and Novozhilov [2] considered the fourth and fifth levels of the indicated structures, respectively. Panin [23] distinguishes four structural levels — from a macrolevel to a microlevel through meso-II and meso-I levels. The characteristic linear sizes r_i and damage parameters k_i/n_i are determined by methods of metal physics for each structural level (for example, using optical and electron microscopy, X-ray methods, etc.) [24]. The force parameters $\sigma_m^{(i)}$ or $\tau_m^{(i)}$ for each structural level can be treated as follows: plastic flow stresses of metals for the macrolevel, plastic flow stresses of the corresponding mesostructures obtained by microindentation [25, 26] for the mesoleveles, and theoretical tensile and shear strengths of ideal crystalline solids, respectively, for the microlevel [2, 4, 7, 27].

The following procedure is proposed to determine the critical generalized SIF $K_{{\rm I}c}$ at the macrolevel $i = 1$ (the SIF $K_{{\rm II}c}$ is determined in a similar manner). For long cracks, from relation (9) we obtain

$$\frac{2K_{{\rm I}c}^0}{\sigma_\infty\sqrt{2\pi n r_1}} = \frac{\sigma_m}{\sigma_\infty}\,\frac{k}{n}, \qquad \text{whence} \quad \frac{\sqrt{2}K_{{\rm I}c}^0}{\sqrt{\pi r_1}} = \frac{k}{\sqrt{n}}\,\sigma_m. \tag{13}$$

For real materials, $k = 1, 2$ and $n = 1, 2, 3$, and, hence, the ratio $k/\sqrt{n}$ is in the range $0.71 \le k/\sqrt{n} \le 1.15$, i.e., it is close to unity. For simplicity, we confine ourselves to the case of $n = k = 1$ (material without damage). From (13), we obtain the theoretical strength of the material of the macrostructure $i = 1$: $\sigma_m = \sqrt{2}K_{{\rm I}c}^0/\sqrt{\pi r_1}$. The value of σ_m is used to calculate the critical generalized SIF $K_{{\rm I}c}$ of a V-shaped notch. For a long V-shaped notch, from relation (7), we obtain

$$\frac{K_{{\rm I}c}}{\lambda_1\sqrt{2\pi}\,r_1^{1-\lambda_1}} = \sigma_m, \qquad \text{whence} \quad K_{{\rm I}c} = \sigma_m\lambda_1\sqrt{2\pi}\,r_1^{1-\lambda_1}. \tag{14}$$

Substitution of (13) into (14) yields the following relation between the critical generalized SIF and the critical SIF of a sharp crack:

$$K_{{\rm I}c} = 2K_{{\rm I}c}^0\lambda_1 r_1^{1/2-\lambda_1}. \tag{15}$$

In the limit as $\alpha \to \pi$, we have $K_{{\rm I}c} \to K_{{\rm I}c}^0$.

Numerical Example. As an example of determining the generalized SIF, we consider three-point bending of a prismatic specimen with a V-shaped notch. The geometry of the specimen and the acting loads are shown in Fig. 4a. We use the following dimensionless geometrical and loading parameters: $t = 1$, $b = 10$, $L = 4b$, and $P = 1$ (t is the specimen thickness). In the calculations, the tip angle was $\beta = 30°$, $45°$, $60°$, $90°$, $120°$, and $150°$ and the notch depth measured in the units of l/b was varied from 0.1 to 0.9. Similar prismatic specimens have been used in toughness studies [28]. Experimental results on three-point bending of polymethylmethacrylate

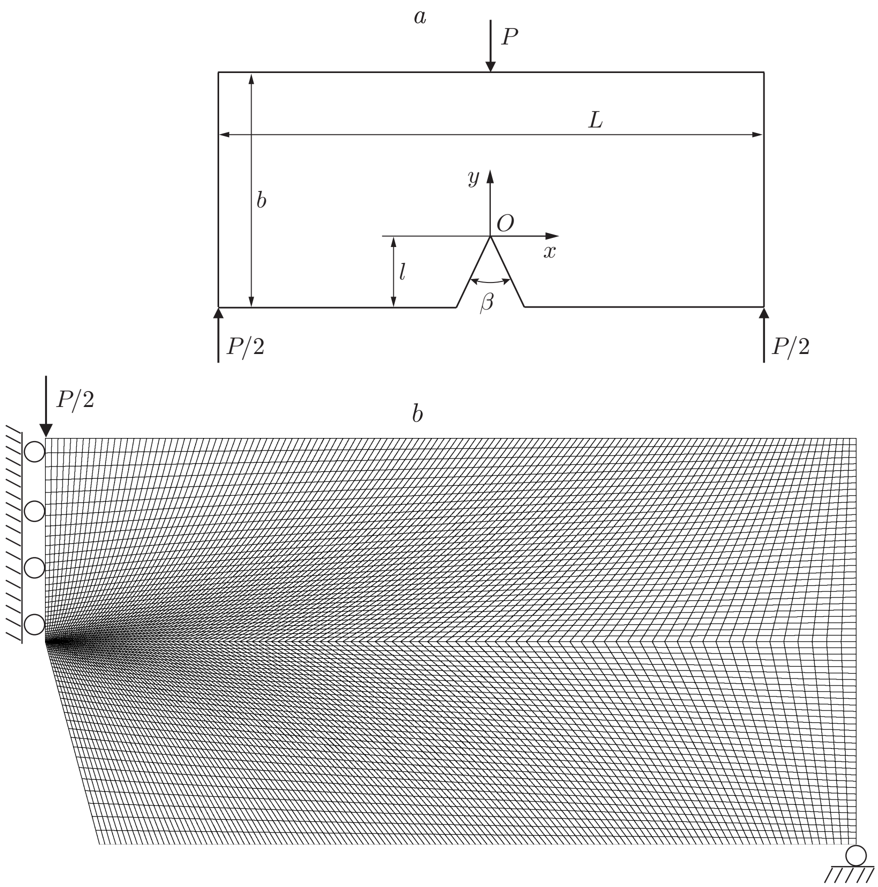

Fig. 4

specimens for various values of the angle β and notch depth were obtained in the experiments of [17], which were aimed at determining the critical SIF and fracture load. In the same study, the stress distribution in the vicinity of a V-shaped notch was obtained by the finite-element method and $K_{\rm I}$ was calculated for the given geometry and loading.

Using the finite-element method, we determined the plane stress distribution in a notched specimen. Figure 4b shows the finite-element mesh for $\beta = 30^\circ$ and boundary conditions. The coordinate origin is located at the notch tip, the Ox axis is horizontal, and the Oy axis is vertical. In the vicinity of the notch tip, the mesh was extremely fine to determine $K_{\rm I}$ as accurately as possible. The mesh shown in Fig. 4b contains 8580 sixteen-node elements connected at 77,809 nodal points. The stresses were calculated at the Gauss–Legendre integration points of the elements adjacent to the Oy axis. A 4×4 integration scheme was used. The generalized SIF $K_{\rm I}$ was calculated by the least-squares method from the relation $\sigma_x(0, y) = K_{\rm I} y^{\lambda_1 - 1}$, where λ_1 is a root of the characteristic equation (3). The numerical experiments showed that it suffices to use the values of σ_x at four points of the element adjacent to the notch tip: the addition of the second and third elements does not improve the calculation accuracy.

The numerical data were compared with analytical solutions. For $\beta = 0^\circ$ (a beam with a crack), an approximation formula for $K_{\rm I}$ is given in [21, p. 360]. Compared to the exact value, the error of the calculated $K_{\rm I}$ is 1.5%.

Figure 5 gives an approximation relation; the points refer to the numerical solution. The values of λ_1 and the dimensionless SIF $K_{\rm I}$ for unit load are listed in Table 1 for various values of β for $l/b = 0.5$. Since the problem is linear, one can easily calculate $K_{\rm I}$ for any other value of P and, using the critical SIF $K_{{\rm I}c}$, obtain the rupture load P_c. Figure 6 shows $K_{\rm I}$ versus notch depth for $\beta = 30^\circ$ (curve 1) and $\beta = 90^\circ$ (curve 2).

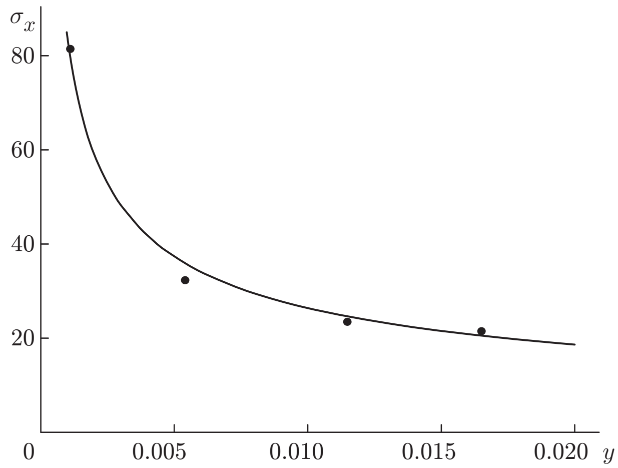

Fig. 5

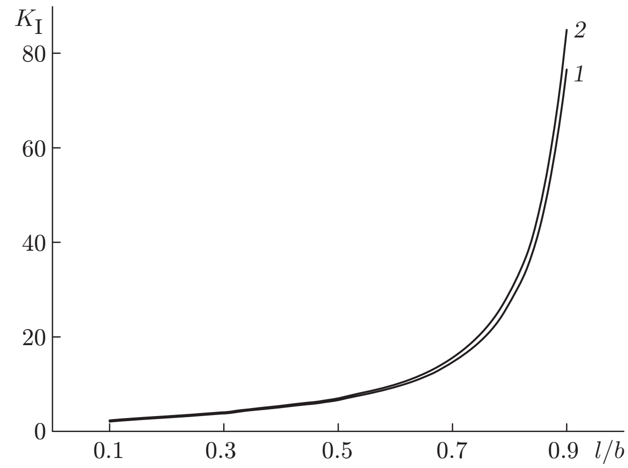

Fig. 6

TABLE 1

β, deg	λ_1	$K_{\rm I}$
0	0.5000	6.6141
30	0.5015	6.6332
45	0.5050	6.6645
60	0.5122	6.7284
90	0.5445	7.0134
120	0.6157	7.5936
150	0.7520	8.6791

Owing to the symmetry of the problem, the shear stresses vanish on the Oy axis, and only $K_{\rm I}$ was determined. For nonsymmetric bending, both tensile and shear stresses occur near the notch; therefore, one should find the generalized SIFs $K_{\rm I}$ and $K_{\rm II}$ using the approximation relation

$$\sigma_x(0,y) = K_{\rm I} y^{\lambda_1 - 1} + K_{\rm II} y^{\lambda_2 - 1}.$$

Conclusions. We pay attention to the dimensionality of the generalized SIFs $K_{\rm I}$ and $K_{\rm II}$, which, as follows from formulas (7) and (8), depends on the tip angle of the notch. In classical fracture mechanics, the critical generalized SIFs of a material $K_{{\rm I}c}$ and $K_{{\rm II}c}$ depend on the tip angle of the notch. According to the discrete integral criteria (5) and (6), the critical state of a crystal structure ahead of the tip of a crack or cut occurs when the averaged stresses in the interval $(0, nr_e)$ reach the theoretical strength with allowance for the damage to the material. Criteria (5) and (6) and the criterion proposed in [12] are force one-parameter criteria in the interval $(0, nr_e)$. These parameters are the theoretical strengths σ_m and τ_m. The minimum length of the averaging interval is equal to r_e. Estimates (7) and (8), which take into account the material structure, are local estimates and are determined mainly by the singularity coefficients $1 - \lambda_1$ and $1 - \lambda_2$.

In classical fracture mechanics, the critical values of the generalized SIFs $K_{{\rm I}c} = K_{{\rm I}c}(\alpha)$ and $K_{{\rm II}c} = K_{{\rm II}c}(\alpha)$ need to be determined for each angle α and each material. It is more reasonable to assume that the critical SIF for a crack is not a material constant. The theoretical strength σ_m or τ_m of the regular structure considered is a material constant, which is seen from formulas (7) and (8), and the generalized SIFs $K_{\rm I}$ and $K_{\rm II}$ constructed for the specified boundary conditions are a convenient approximation of solutions (1) and (2).

Thus, for all three types of stresses (tension, shear, and antiplane strain), we obtain one equation that relates the critical SIF to the theoretical strength of the single crystal:

$$\frac{K_{ic}}{\lambda_i \sqrt{2\pi}\,(nr_e)^{1-\lambda_i}}\,\frac{1}{\sigma_\infty^i} = \frac{\sigma_m^i}{\sigma_\infty^i}\,\frac{k}{n} - 1,$$

where superscripts i = I, II, and III refer to three types of cracks; $\sigma_m^{\rm I} = \sigma_m$ and $\sigma_m^{\rm II} = \tau_m$ (shear), $\sigma_m^{\rm III} = \tau_m$ (antiplane strain), $\sigma_\infty^{\rm I} = \sigma_\infty$ and $\sigma_\infty^{\rm II} = \tau_\infty$ (shear), and $\sigma_\infty^{\rm III} = \tau_\infty$ (antiplane strain); $\lambda_{\rm I} = \lambda_1$ is a root of the characteristic equation (3), $\lambda_{\rm II} = \lambda_2$ is a root of the characteristic equation (4), and $\lambda_{\rm III} = \omega$ is a root of the characteristic equation (1.2) of [20].

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